Related Rate Problems

Rate - Slope - derivatives

8 m/s mi/h ft/s

km/h newtons/m = N/m

Rates

$$\frac{dA}{dt} \qquad \frac{dV}{dt}$$

$$\frac{dw}{dt} \qquad \frac{dr}{dt}$$

With respect to time

$\frac{dy}{dx}$ deriv of y with respect to x.

$\frac{du}{dx}$ der. of u with respect to x.

Write an equation that relates

$\frac{dA}{dt}$ and $\frac{dr}{dt}$.

① $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

$V = \pi r^2 h$ → product rule

$$\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt} + h \cdot 2\pi r \cdot \frac{dr}{dt}$$

$\frac{dV}{dt} = 10\ m^3/hr$ $\qquad V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = ?$ when (at what instant) $r = 5$ $\qquad \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$10 = 4\pi \cdot 25 \cdot \frac{dr}{dt}$

$\frac{10}{100\pi} = \frac{dr}{dt}$

$\frac{1}{10\pi} = \frac{dr}{dt}$

Our radius is changing at a rate of $\frac{1}{10\pi}$ m/hr.

$\frac{dV}{dt} = 100 ft^3/min$ $V = \frac{1}{3}\pi r^2 h$

$\frac{dh}{dt}$? when $h = 10ft.$

$r = h$

$V = \frac{1}{3}\pi h^3$

$\frac{dV}{dt} = \pi h^2 \cdot \frac{dh}{dt}$

$100 = \pi \cdot 10^2 \frac{dh}{dt}$

$\frac{100}{100\pi} = \frac{dh}{dt}$

$\frac{1}{\pi} = \frac{dh}{dt}$

My height is changing at a rate of $\frac{1}{\pi}$ ft/min.

#3

$V = x^3$

$\frac{dx}{dt} = 3 \text{ cm/sec.}$

$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

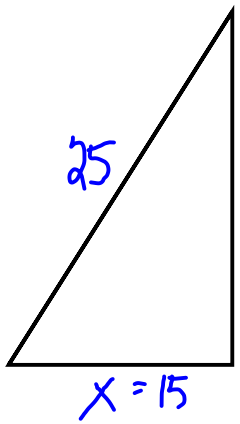
$\frac{dV}{dt} = ?$ when $x = 10 \text{ cm}$

$\frac{dV}{dt} = 3 \cdot 10^2 \cdot 3$

$\frac{dV}{dt} = 900$

Our volume of the cube is changing at a rate of $900 \text{ cm}^3/\text{sec.}$

#4.



$\frac{dx}{dt} = 2 ft/s$

$\frac{dy}{dt} = ?$ when $x=15$

$x^2 + y^2 = 625$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$2 \cdot 15 \cdot 2 + 2 \cdot 20 \cdot \frac{dy}{dt} = 0$

$60 + 40\frac{dy}{dt} = 0$

$40\frac{dy}{dt} = -60$

$\frac{dy}{dt} = \frac{-60}{40}$

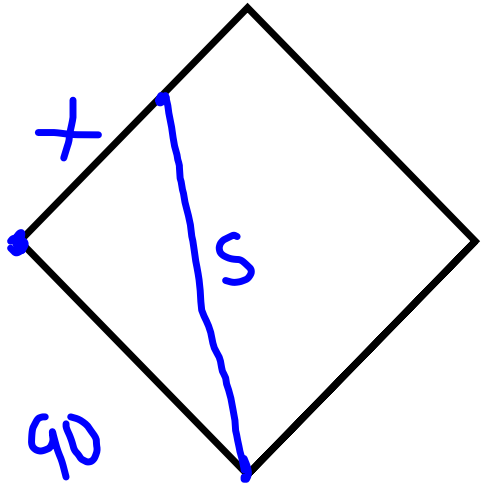
$\frac{dy}{dt} = -\frac{3}{2}$

The top of the ladder is moving down the wall at a rate of $\frac{3}{2}$ ft/sec.

neg.

HW p. 237-238

8, 9, 11, 12



$x^2 + 90^2 = s^2$

$\frac{dx}{dt} = 2 ft/s$ at $x = 30$

$\frac{ds}{dt} = ?$